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A Fundamental of Scientific Inquiry in Economics

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A FUNDAMENTAL OF SCIENTIFIC INQUIRY IN ECONOMICS

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ABSTRACT

This article will fundamentally change the way economists look at or understand mathematics, thus it would prevent them from developing false economic theories in the future. It will also identify the false economic theories developed in the past. Economists have manipulated mathematics, substituting one equation for another equation in order to build economic theories, not knowing what mathematics was doing or not knowing the limitation of mathematics in terms of explaining the behavior of economic units. I will present two different types of interpreting a mathematical equation: (A) Interpreting a variable, (which has a functional relationship such as dy/dx) in terms of a constant number, and (B) Interpretation of a constant number in terms of a variable. The former, (A), is proven to represent universally true facts, but the latter, (B), is proven to represent inconclusive facts or to represent facts, which are not universally true. Thus, any economic theory developed based on (B) will have inconclusive

statement; yet economists have not noticed this fallacy thus continue to build and interpret economic theories as if they were universally true fact. The Slutsky equation is one of them, which I will discuss in this paper. However, this paper is not intended to extensively survey false economic theories developed based on (B) in the past

I. INTRODUCTION

Mathematics has made significant contributions to the development of economics theories in the past. An emphasis on the use of mathematics has been extensive in economics. However, the assumption, which underlies the use of mathematics, has not been formerly scrutinized. Consequently, errors in the development of economic theories have been unnoticed and misleading theories have been established in economic science. The objective of this article is to develop a model, which can test whether or not an application of mathematics into an economic theory is valid, thus putting mathematics in a proper perspective in economic science. In the past, mathematics has been applied in developing economic theories without questioning if an application of mathematics into economic theories is valid or not.

Specifically, I will develop a model, which can test whether or not one equation can be substituted into another equation in the process of development of an economic theory. In economic science (or behavioral science), when the substitution of an equation is carried out in algebraic manipulation, the meaning of an equation is also substituted along with the mathematical substitution or the interpretation of an equation must go with it on algebraic manipulation. Then we have to make sure that the meaning or the interpretation of an equation (which goes with it on algebraic manipulation) would indicate the universally true fact, not having any inconclusive fact.[1] But if it is purely mathematics, the meaning of an equation to be substituted is a trivial issue or it can be ignored or algebraic manipulation can be carried out by ignoring its meaning. This is the difference between economics and mathematics as far as the application of mathematics into economics is concerned. In developing an economic theory, this difference has never been scrutinized in the past whenever mathematics is applied in economics.

[1] In the past, the meaning of an equation, which is not universally true fact went along with mathematical substitution, thus violating the rule for usage of the mathematical equality sign and established false economic theories. We will discuss this in detail later.

II. INTERPRETATION OF ALGEBRAIC MANIPULATION

Mathematics has been used in the past in an economic theory without the development of a criterion that can test whether or not its usage is justifiable in developing an economic theory. In other words, mathematics has been used in the past in the development of an economic theory without establishment of a formal agreement about the way in which the mathematical equation is interpreted. Thus, in this paper, I will outline formally and establish the way in which an equation (to be substituted in the algebraic manipulation) should be interpreted in economics. In other words, I will examine the way in which a mathematical equation is interpreted, and study its implication to algebraic manipulation. For this purpose, I will present two types of the interpretation of an equation as shown below so that the validity of the substitution of an equation can be tested before we carry out the algebraic manipulation:

(A): The algebraic definition of interpretational direction A, hereafter called the interpretational direction A or simply called (A).

(B) The algebraic definition of interpretational direction B, hereafter called the interpretational direction B or simply called (B).

The first one (A) represents the type of substitution of an equation in which a variable is replaced by a constant number in the process of substituting an equation. The second one (B) represents the type of substitution of an equation in which a constant number is replaced by a variable. Here is an example of an equation to explain the above (A) and (B):

$$dy/dx = 5 \qquad (1)$$

There are two elements in (1); one is mathematics and the other is interpretation. Mathematically, (1) indicates that two quantities on the left hand side of the equation and the right hand side of the equation are the same, thus they are universally true or the equality sign indicates the universally true fact that the both sides are equal. However, when we interpret the meaning of dy/dx and 5 in (1), the term on the left hand side of the equation, which is a variable dy/dx, and the term on the right hand side of the equation which is a constant number 5, are not always the same**.** I will elaborate this point in detail by presenting two definitions of an interpretation of an equation mentioned above as (A) and (B).

An interpretation of an equation is defined as “the algebraic definition of interpretational direction A” to indicate that the meaning of a variable such as dy/dx in (1) is interpreted in terms of a constant number such as 5 in (1). On the other hand, an interpretation of an equation is defined as "the algebraic definition of interpretational direction B" to indicate that the meaning of a constant number such as 5 in (1) is interpreted in terms of a variable such as dy/dx in (1).

One of the important objectives of this article is to reveal that the above mentioned (A) and (B) are not the same. Stated specifically, (A) is always acceptable or universally true, thus the equality sign can be used just like mathematics can use the equality sign, but (B) is not always acceptable or (B) is not universally true; thus, the equality sign can not be used as mathematics use the equality sign. (In the past, (B) has been always accepted as universally true fact; thus, the equality sign has been used in applying mathematics in economics as evidenced in "Comparative Static Analysis." This is obviously wrong according the theory developed in this paper: For example, the Slutsky equation has been derived based on (B); thus, it is a false theory as we will reveal in this paper later).

I will begin this by first explaining (A). The interpretation of a variable in an equation in terms of a constant number means that the meaning of a variable is interpreted or summarized by a constant number. For example, dy/dx in (1) means the amount of change in y resulting from an additional unit of change in x. This meaning can be summarized numerically in (1) as five units or expressed fully as the amount of change in y resulting from one additional units of change in x is equal to five units. In this case, the equality sign is used correctly because the interpretation of the equation is done correctly or the interpretation of the equation is universally true (and there is no contradiction nor inconsistency), and it is consistent with the way in which mathematics uses the equality sign. *Mathematically*, this interpretation or *summary* means that the meaning of a variable can be replaced by a constant number, or the former statement can be replaced by the latter statement in interpreting the equation. Although a constant number such as 5 in (1) does not have any specific meaning, it can be used to summarize the meaning of a variable.

Next, I will scrutinize (B), which intends to interpret the meaning of a constant number in an equation in terms of a variable. The interpretation of a constant number in an equation in terms of a variable means that the meaning of a constant number can be summarized by a variable. For example, the meaning of a constant number such as 5 in (1) can be summarized by a variable such as dy/dx in (1). In other words, according to (B), a constant number such as 5 in (1) indicates that there is a functional relationship stated in a variable such as dy/dx in (1). *Mathematically, the* *above summary* means that the meaning of a constant number can be replaced by a variable. Stated alternatively it indicates that the meaning of a constant number such as 5 in (1) can be replaced by the meaning a variable such as dy/dx in (1). Obviously, this interpretation of an equation is wrong since a constant number itself does not have any specific meaning. Thus the equality sign cannot be used since its interpretation of an equation is not universally true nor conclusive. To put it simply, if the equality sign is used to carry out substitution of an equation under the interpretational direction B, it is wrong or will have inconclusive conclusion. *(Mathematical substitution or derivation cannot beget a predicable relationship or a functional relation, although mathematical derivation can be used to verify the existence of a predicable relation. We cannot rely on mathematical derivation to create a predicable relation: we must create assumption to create the predicable relation.)* Next, I will discuss further in the next section the reason why the interpretational direction B does not make sense.

III. A THEORY OF THE USAGE OF LANGUAGE

In this section, we will investigate a usage of language so that the meaning of interpretation of an equation can be clearly understood. For this purpose, we will distinguish the meaning of the specific and the general concepts in a statement. A specific concept in a statement indicates a specific meaning, which rule out possibilities of including other meanings. On the other hand, a general concept indicates a general meaning, which could include possibilities of many other meanings beside what it says. For example, Mr. B. Smith is a man who is tall. In the above statement, “Mr. B. Smith” is a specific concept, while “a man who is tall” is a general concept. The former is specific because there is only one Mr. B. Smith in discussion. The latter is general because there are many other men who are tall beside Mr. B. Smith. For another example, U.S. GNP growth rate is 3%. “U.S. GNP growth rate” is specific because it indicates only one specific meaning in the statement or it does not leave a possibility of meaning other things. On the other hand, “3%” is general because 3% could mean many other things. Note that a constant number is an extreme end of general concept or an example of the most general concept since a constant number such as 3 or -2 could mean almost anything in the world.

Having distinguished the difference between specific and general, we will clarify the rule under which specific and general are used in constructing a statement. The usage of language dictates in constructing a statement that we can explain a specific concept in terms of a general concept or summarize the meaning of a specific concept by the meaning of a general concept, but cannot explain a general concept in terms of a specific concept or cannot summarize the meaning of a general concept by the meaning of a specific concept._ For example, Mr. Jones is (the man who is) 6 feet (tall), where "Mr. Jones" is a specific concept and "(the man who is) 6 feet (tall)" is a general concept. This is a statement in which a specific concept is explained or summarized in terms of a general concept. This is true statement and this kind of statement is acceptable. In other words, a statement that explains a specific concept in terms of a general concept is acceptable because it is true statement or universally true. This is an example of the interpretational direction of A.

On the other hand, if we say, "(man who is) 6 feet (tall) is Mr. Jones," it is not universally true statement. It is an example of a statement in which a general concept is explained in terms of a specific concept. This is because 6 feet (tall) is a general concept and Mr. Jones is specific concept. There are many other people who are 6 feet tall, some of whom we know and also other people whom we do not know. Thus the statement, which says, "6 feet tall is Mr. Jones" is not universally true fact and such statement is not acceptable. This is the interpretational direction B.

Now, we will apply the above stated rule of the usage of language to an interpretation of an equation. If we interpret a mathematical equation in which there is a variable at the left side of the equation and a constant number at the right side of the equation such as shown in (1), there are two methods of interpreting the equation. A variable in an equation such as dy/dx in (1) is a specific concept since it has a specific meaning stated in terms of a functional relation between x and y, where a constant number in an equation such as 5 in (1) is a general concept.

First, the method of interpreting an equation, in which a variable is explained in terms of a constant number, is equivalent to explaining a specific concept in terms of a general concept. This method of interpreting an equation is, by definition, the interpretational direction A and it is an acceptable interpretation of a statement according to the rule of the usage of language discussed above.

On the other hand, the method of interpreting an equation, in which a constant number is explained in terms of a variable, is equivalent to explaining a general concept in terms of a specific concept. This method of interpreting an equation, by definition, the interpretational direction B and it is an unacceptable interpretation of a statement according to the rule of the usage of language explained above because its statement is not conclusive nor universally true. The equality sign is used only to state a universally true fact or conclusive fact without any exception. The statement explained based on Interpretational direction B is not conclusive, which means that there are some other reasons besides what that statement says. For example, as we discussed before, when we say, "6 feet is Mr. Jones," is not universally true statement. This is because there are many other people who are also 6 feet tall, although it is true that Mr. Jones is 6 feet tall.

Thus, any economic theory, developed by substituting an equation into another equation based on the interpretational direction B, indicates what is false or inconclusive statement. This is because the meaning of an equation substituted into another equation is not universally true fact. (Or stated alternatively, it is not conclusive; thus, it is false in relationship to the use of the mathematical equality sign because what is false or inconclusive statement or interpretation was substituted into another equation when an equation was substituted.) It is false use of the mathematical equality sign. Thus when an equation was substituted, an inconclusive statement (which is not universally true fact) went with it while algebraic manipulation was carried out. We have to make sure that only such an equation, which indicates universally true fact, should be substituted into another equation when we carry out algebraic manipulation.

I will reinstate this view again here. In mathematics, the use of the substitution principle requires: (1) Quantities of the both sides of an equation must be equal or the same so that the relation stated in mathematical equality sign is universally true or it would be true without any exception. However, in economics when mathematics is applied, the substitution principle requires: (2) additionally (in addition to (1)) that the meaning of the both sides of an equation (or the meaning expressed in the equation) must be also universally true or consistent. Either that or that the meaning expressed by the both side of the equation has to be universally true, which means that such meaning is true without any exception. This means that when the substitution of an equation is carried out in developing an economic theory, we should not permit the execution the substitution of an equation into another equation if such work is done based on Interpretational Direction B. Unfortunately, the Slutsky equation is derived based on (B); thus, its mathematical derivation was the misuse of the mathematical equality sign. It is proven in this paper that the interpretational direction B is false or not conclusive that one cannot use mathematical equality sign. It is imperative to have a clear understanding of the meaning of mathematical equality sign when we use mathematics in developing an economic theory.

IV. SLUTSKY EQUATION

We have clearly shown that any economic theory developed based on the interpretational direction B is false, and I have given enough explanation why such economic theories are false. The Slutsky equation is one of the examples of false economic theories developed based on the interpretational direction B. In this section, I will explain additional specific reasons why the Slutsky equation states inconclusive and false facts, although such additional explanation might not be necessary, since full theory was given to illustrate why the interpretational direction B is wrong.

To begin, I will reinstate the hypothesis, upon which mathematical equality sign is used. In applying mathematics to development of an economic theory, the use of the substitution principle require, in economics, that two conditions must be met: (1) Quantity of both sides of an equation must be the same or true, and (2) the meaning of the both sides of an equation must be the same or universally true. However, in conducting, the substitution principle, mathematics requires only the first one (1). So if we use mathematics in developing an economic theory, we tend to ignore the (2) or we are overly preoccupied with (1) and forget (2), while we are building an economic theory. This might have caused an error in building mathematical model in economics, although it might be correct in mathematics.

First, I will review briefly the way in which the Slutsky equation is derived and show how the interpretational direction B has been used in its derivation before I discuss a fact that the conclusion of the Slutsky equation contains inconsistencies and contradictions in its interpretation. For this purpose, the commonly known Slutsky equation is shown below in terms of two goods.[2]

(4a) $\delta q_1/\delta p_1 = (\delta q_1/\delta p_1)\ du = 0 \qquad -q_1(\delta q_1/\delta y)\ dp_i \qquad i = 1, 2$

In order to demonstrate how the Slutsky equation is derived, the following equations are provided below.

(4b) $\delta q_1/\delta p_1 \quad = \quad (D_{11}\lambda/D) \quad + \quad q_1\ (D_{31}/D)$

(4c) $(\delta q_1/\delta y)\ dp_i = 0 \quad = \quad -\ (D_{31}/D)$

(4d) $(\delta q_1/\delta p_1)\ du = 0 \quad = \quad (D_{11}\lambda/D)$

As it is well known, (4c) and (4d) are substituted into (4b) in order to derive (4a), which we have called the Slutsky equation. We will reveal that the

[2] For a multi good case, see P. A. Samuelson, Chapter 5

above substitution of equations to derive (4a) and subsequent interpretation of (4a) have resulted in the use of the algebraic definition of interpretational direction B.

We will examine the above substitution problem in detail. $(D_{11}\lambda/D)$ in (4b) and (4d) is a constant number such as 5 or -7. The substitution of (4d) into (4b) means that we replaced $(D_{11}\ l/D)$ in (4b) by $(\delta q_1/\delta p_1)$ du = 0 from (4d) or replaced a constant number by a variable. By analogy (D_{31}/D) in (4c) is a constant number. When we substitute (4c) into (4b), we replace (D_{31}/D) in (4b) by $-(\delta q_1/\delta y)\ dp_i = 0$ from (4c) or replace a constant number by a variable. Therefore, if equation (4a) is interpreted, it is the interpretational direction B. Thus it was the misuse of mathematics.

Any theory developed based in the interpretation direction B is bound to have false conclusion. As we mentioned in our example, which says, "6 feet is Mr. Jones is," the statement derived based on the interpretational direction B. Thus this cannot be a universally true statement because there are many other people who are 6 feet tall. In the same way, the substation effect in the Slutsky equation says, "An increase in the quantity of x from x_1 to x_2 is only due to the change in price of x," as illustrated in the Figure 1. Such statement is not universally true, thus false because there are some other things which cause an increase in the quantity of x from x_1 to x_2 other than the change in the price of x (just like there are some other people who are 6 feet tall other than Mr. Jones). This happens because the substitution effect in the Slutsky equation is derived based on the interpretation direction B.

I will explain this in detail below. The crucial assumption made in formulating and interpreting the substitution effect is that a consumer's income is being compensated or varied as implied by du = 0 in (4d) (in order to keep the consumer's real income constant or keep the consumer on the same indifference curve).[3] The above assumption is contradicting the meaning of the partial derivative, $(\delta q_1/\delta p_1)$ in interpreting substitution effect in (4d) for the following reason. The partial derivative $(\delta q_1/\delta p_1)du=0$ in (4d) examines the effect of a change in the price of a product excluding all other effects. The last part of the above statement, "excluding all other effect," is in contradiction with the assumption in (4d), du = 0, which implies that the consumer's income is compensated.[4] Therefore, if the substitution effect in the Slutsky equation shown in (4d) is correctly interpreted, the change in quantity of a product demanded in (4d) is realized not only due to a change

[3] See E. Malinvaud (1972), pp. 34-39.
[4] The interpretation of (4d) should include both the meaning of partial derivative, $(\delta q_1/\delta p_1)$ and the meaning of its assumption, du = 0, which implies consumer's income is compensated. The former meaning indicates that all other effects are kept constant which implies consumer's income is kept constant, while the latter meaning indicates consumer's income is compensated or varied. The above two meanings are contradicting each other.

in the price of a product (which is indicated by its partial derivative) but also due to the income compensated or the change in income or budget (which is implied by its assumption, du=0). However, the latter has been ignored in interpreting the substitution effect.

While income is compensated, something happened to the budget or income of the consumer. How much income is compensated is never mentioned and it is important that the amount of income compensated to be exact such that two budget lines on the original indifference curves had to the same. This seldom happens as we see in the Figure 1 below (unless point b happened to be at point e and that can not happen.)

This is illustrated in Figure 1. In order to keep budget line on the same indifference curve, two budget lines are tangential to the one indifference curve. The budget line AB and CD are not the same. (It is impossible for the budget line ad and cd to be the same. It can only happen by accident.) Thus, when budget line AB is changed to budget line CD, the consumer's budget increases (or decreases) and that change in the budget is causing also the change in the quantity of x when the price of x is changed. The substitution effect is summarized as an increase in the quantity demanded from x_1 to x_2. This increased quantity is not only purely due to the change in the price of x, but also due to the change in budget (and also importantly may be due to something else which we do not know at this moment.) This is the kind of a problem that appears when an equation is substituted based on the interpretational direction B as evidenced in the substation effect in the Slutsky equation. We discussed this kind of problem with an example before by saying, “6 feet is Mr. Jones,” is not universally true statement.

Another contradiction in the Slutsky equation is found in the inducement factor in explaining income effect, which is shown as the increase in the quantity of x from x_2 to x_3 in Figure 1. The income effect is supposed to demonstrate the effect of a change in real income (or money income when prices are kept constant), while keeping prices of all the products constant as indicated by $dp_{ii} = 0$ in (4c). The contradiction in the above statement is that if all prices of the products are kept constant in demonstrating the income effect, there is nothing in the model, which explains why a change in real income should take place, which is supposed to induce the income effect. The truth of the matter is that there is nothing in the model, which explains why (real) income should ever change when all the prices of products are kept constant. Again, such contradiction appeared because the income effect in the Slutsky equation was derived based on the interpretational direction B. This has been a puzzle and a similar concern has existed in the theory of inferior factors.[5]

V. CONCLUSIONS

Over one hundred years, the use of mathematics in developing economic theories has become explosive. Mathematics has become the root of the production of fruits by creating countless economic theories. They became a society of technicians, not that of engineers nor that of inspiration or economic insight, since anybody who knows how to mange the technique of mathematics becomes in popular demand. The people who developed an overwhelming number of economics theories in the past did not understand that some of the roots of the theories were rotten. The fruits may look beautiful on the surface but such fruits are bound to be rotten if their roots are rotten. The rotten roots mean that the economic theories based on mathematical work have inconclusive or contradicting conclusions. But economists have not questioned such inconclusive conclusions; in stead, they made up what seems to be beautiful interpretation of the theories because they did not know the roots were rotten. Thus, they forced to come

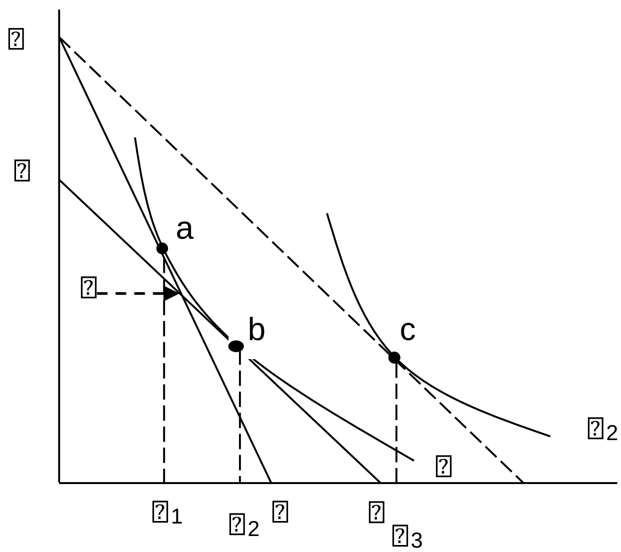
[5] See C. E. Ferguson and T. R. Saving. The question raised was the direction of change in output after a change in a factor. However, the answer to the question is ironically given in terms of change in output or expenditure while keeping all factor prices constant as show in n1c. The truth of the fact is that there is nothing in the model that explains why output or expenditure should ever change when factor prices do not change.

up with an artificial interpretation of the conclusions of the theories because they believed their roots were true and right or healthy, not rotten.

All of such problems appeared in developing economics theories because the economists did not understand the meaning of the equality sign when mathematics is applied in economics or did not know the difference between mathematics and economics in terms of the use of the mathematical equality sign. I have provided this new insight about the way in which we use the equality sign between two branches of the sciences mentioned above. Finally, I have shown in this paper how rotten roots would be planted in building economic theories, when we substitute an equation to another equation based on "the interpretational direction B."

In a society of technicians or in today's society, any mathematical work in economics is accepted or welcomed as a contribution to the field of economics, even if its roots are rotten. But if someone discovers how the rotten roots can be planted in building economic theories because of the interpretational direction B, no one else is willing to believe it is the contribution to the existing field of economics theories. Yet, I believe that it is time to examine the study, which welcomes inspiration or search for new insight and it must be welcomed as a significant contribution to the field of economics so as to clean up all the rotten roots and fruits.

Figure I



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